On the Generalized Correspondence Principle

Joseph Manjavidze [1]

Аннотация

A generalization of the correspondence principle is discussed in the way being independent of the value of quantum corrections. This approach leads to a converging quantum field theory as soon as the corresponding symmetry constraints are taken into account. The obtained quantum field theory with symmetry does not contain particle-like excitations.

1. The aim of the article is to allow readership to take a look at the generalized correspondence principle (GCP): *if a classical analog exists then the quantum process probability is defined by equilibrium among ordinary mechanical forces and quantum force $\hbar j$ of the arbitrary strength* [1][2]. Notice that GCP is close to the principle of d'Alembert: *the dynamics of the system is defined by equilibrium of all acting in it forces, if the motion is time reversible* [2]. The d'Alembert principle was formulated in 1743, long before the quantum mechanics.

At the same time GCP differs from the well known Bohr correspondence principle offered in 1923 which runs that *the quantum mechanical systems behavior can be described by classical mechanics laws in the limit of large quantum numbers* [3]. The role of the Bohr correspondence principle can not be overestimated. It introduces the Newton laws into the quantum mechanics: if there is a classical analog in the system then the *in*-state obtains the exponential factor $e^{iS/\hbar}$ during its transition into the *out*-state according to the general conclusion of P. Dirac [4]. It is important here that S have the structure of the classical action.

Let us consider a particle collision process. The point is that the quantum nature of collision leads to the perturbation of the object in question as well as of the incident particle. One of the possible scenario of the process may be marked by the field $\varphi(x)$, where x is the 4-coordinate in Minkowski metric, $x^2 = t^2 - \mathbf{x}^2$. This gives the factor $e^{iS(\varphi)/\hbar}$ to each configuration of φ. Therefore quantum nature of the problem means the necessity to take into account all configurations, i.e. to find the sum (integral) over φ [5],

$$A = \Sigma_{\{\varphi\}} e^{iS(\varphi)/\hbar}.$$

The definition of set $\{\varphi\}$ is a fundamental problem of quantum theories and this problem solves GCP. The Bohr correspondence principle partly solves it: the variation of $S(\varphi)$ over φ must be equal to zero,

$$\frac{\delta S(\varphi)}{\delta \varphi(x)} = 0. \tag{1}$$

That equation simply means that it is desirable to have the minimal oscillations during the transition $(in \rightarrow out)$ state. It is held in quantum case with exponential accuracy and becomes exact only in the classical limit $\hbar = 0$. In terms of d'Alembert Eq. (1) means the equilibrium among ordinary kinetic and potential forces.

---

[1] Joint Institute for Nuclear Research, Joliot-Curie str. 6, 141980, Dubna, Russia; Andronikashvili Institute of Physics, Tbilisi State University, Tamarashvili str. 6, 380077, Tbilisi, Georgia; tel: (+7-49621) 6 35 17, e-mail: joseph@jinr.ru

[2]**Another formulation of GCP also exists:** *if there is a classical analog then for an external observer the quantum system looks like classical which is excited by quantum force.* **This interpretation was offered by A.Sissakian.**

2. The key word for GCP is the probability, R, which is equal to modulo square of amplitude A, $R = |A|^2 = A \cdot A^*$. Notice first of all that only the absorbed wave can be detected by an experiment since the total probability is a conserved quantity, i.e. following the optical theorem, $A \cdot A^* = 2\mathrm{Im}A$, only the imaginary (absorption) part of amplitude, $\mathrm{Im}A$, describes the result of the measurement. Notice also that the product $A \cdot A^*$ describes the time reversible (a closed path) motion since the end points of the motion $(in \to out)$ which is described by A and $(in \leftarrow out)$ described by A^* coincide. As a result the phase of A is canceled in the modulo square $R = |A|^2$. Therefore, the notion of quantum probability contains extra requirements and GCP is their consequence.

Indeed, let us consider a motion of a particle with energy E in a potential hole. Corresponding amplitude looks as follows:

$$A(\mathbf{x}_1, \mathbf{x}_2; E) = \sum_n \frac{\psi_n(\mathbf{x}_1)\psi_n^*(\mathbf{x}_2)}{E - E_n + i\varepsilon},$$

where E_n is the bound state energy. One can note that $A(\mathbf{x}_1, \mathbf{x}_2; E)$ is defined on the whole axis of E. Let us consider now the probability $\rho(E)$ to find the particle with energy E. Taking into account the ortho-normalizability of the wave function ψ_n one can find that

$$\rho(E) = \int d\mathbf{x}_1 d\mathbf{x}_2 |A(\mathbf{x}_1, \mathbf{x}_x; E)|^2 = \sum_n \frac{1}{|E - E_n + i\varepsilon|^2} = \frac{\pi}{\varepsilon} \sum_n \delta(E - E_n). \quad (2)$$

One may conclude that the rightly defined observable contains the above mentioned requirements inherently. It is noticeable also that $\rho(E) \neq 0$ only for physically acceptable values of $E = E_n$.

One can show [1] that (i) if there is a classical analog in the system, (ii) because of the closed-path boundary conditions and (iii) cancelation of $\mathrm{Re}A$, we have:

$$R = \lim_{j=e=0} e^{-i\hat{\mathbb{K}}(j,e)} \int DM e^{iU(\varphi,e)/\hbar}, \quad (3)$$

where $\hat{\mathbb{K}}$ in the first exponent is the perturbations generating operator,

$$2\hat{\mathbb{K}} = \int dx \frac{\delta}{\delta j(x)} \frac{\delta}{\delta e(x)},$$

the interactions are induced by the functional

$$U(\varphi, e) = S(\varphi + e) - S(\varphi - e) - 2 \int dx \frac{\delta S(\varphi)}{\delta \varphi(x)} e(x),$$

where $e = e(x)$ is the auxiliary field which describes the quantum deformations of φ, and the Dirac measure

$$DM = \prod_x d\varphi(x)\delta \left(\frac{\delta S(\varphi)}{\delta \varphi(x)} + \hbar j(x) \right).$$

One can trace the analogy of δ-function in DM with δ-function which has appeared in Eq. (2). Therefore because of the Dirac δ-function DM is defined on the *complete set* of the physics field configurations. The formula (3) describes the vacuum-into-vacuum transition probability. An analogous formula exists in the case of the particle production [6].

But it must be underlined that it is impossible to use (3) for the quantum tunneling effects, when the action S becomes complex quantity[3]. The integral (3) can be analytically continued on the complex time contour $C = C_+ + C_-$, $C_\pm : t \to t \pm i\varepsilon$, $\varepsilon \to +0$, to avoid the singularities if they occur.

It is noticeable that DM is defined by the strict equation (GCP):

$$\frac{\delta S(\varphi)}{\delta \varphi(x)} = -\hbar j(x). \tag{4}$$

It is rightful for arbitrary value of $\hbar$, not only for semiclassical approximation $\hbar \to 0$. Meanwhile the quantum corrections are generated by the given operator $\hat{\mathbb{K}}$.

The reason of Eq. (4) appearance is as follows. The main point of the GCP formalism is cancelation of $\mathrm{Re}A$ if $A \cdot A^*$ is calculated. It becomes possible because of the closed-path prescription which ensures the equilibrium of forces, i.e. the dynamics must be defined by Eq. (1) at $\hbar = 0$ (Bohr). The "additional" quantum force, $\hbar j(x)$, is introduced at $\hbar \neq 0$ to provide equilibrium (d'Alembert). As a result we come to the *exact* Eq. (4).

3. Let us consider now the most evident consequences of GCP. First of all δ-likeness of the measure, DM, orders to use a complete set of strict solutions of Eq. (4) in vicinity of $j = 0$. Notice there is a missing interference among various trajectories.

Next, only one term in the sum over solutions must be taken into account. Indeed, let us consider the simplest example of a particle moving into a potential hole which has a minimum at the origin, $\varphi = 0$. Then Eq. (4) will have two solutions at the classical limit when the source of quantum perturbations $j = 0$: the particle all time rests on bottom of the hole, $\varphi_1 = 0$, and the particle moves with energy ϵ, $\varphi_2 = u(t + t_0, \epsilon)$, where t_0 is a starting time. Turning on $j(t)$ one can find two quantum trajectories.

One must integrate over t_0 and $\epsilon \geq 0$ since one must take into account all trajectories. In this case the contribution of $u(t + t_0, \epsilon)$ is proportional to the infinite integral over t_0, i.e. it is proportional to the volume of the group of time translations, Ω. At the same time the contribution of φ_1 is finite. Therefore, as it follows from the definition of Dirac δ-function, $R = R(\varphi_1) + \Omega R(\varphi_2)$ and the contribution of φ_1, $R(\varphi_1)$, is improbable since it is realized on zero measure: its contribution is $\sim 1/\Omega$ in comparison with the contribution of $\varphi_2 = u$. In other words the contribution of φ_1 occupies a point-like volume in the phase space and as the consequence it must be omitted as it is improbable.

One may conclude that in the situation of general position [8], when there is no external influence, the trajectories which maximally break the symmetry of the action are most probable. The path-integral formalism based on this selection rule[4] becomes self-consistent, it does not require external assumptions and leads to *the theories with symmetry*. One can note that the gauge invariant quantum chromodynamics (QCD), just as quantum electrodynamics (QED), can not be considered as the theory with symmetry since the space-time symmetry stays untouched in it[5].

The measure of contribution in the general case is defined by a number of parameters (η, ξ), such as (ϵ, t_0) in the considered example: $\eta = \epsilon$ and $\xi = t + t_0$ in the semiclassical approximation. They form the space W, $(\eta, \xi) \in W$, and the volume of W, Ω, defines the measure of a given trajectory. Therefore, our selection rule means that the trajectories of the largest dimension W are most probable.

---

[3]The tunneling effects can be described by an analytical continuation of the classical trajectory under the potential barrier [7].

[4]This selection rule has not been used in the ordinary formulations of quantum theories but it is widely known in classical mechanics, see for example formulation of Kolmogorov-Arnold-Mozer theorem in the textbook [8].

[5]The gauge degrees of freedom play no role here since an arbitrary solution of Yang-Mills equation breaks it.

4. Another important property of GCP is a possibility to perform the transformation of integrants, i.e. to use the most powerful method of classical mechanics. This becomes possible due to the δ-likeness of the measure: a transformed measure will be again δ-like which ensures conservation of the total probability. A naive transformation of variables in the amplitude is cumbersome and usually produces bad results [9] due to stochastic nature of quantum fluctuations [10].

The transformations can be used to formulate the quantization of non-linear wave u in terms of the generalized coordinates[6].

The first question is: how can "the generalized coordinates" be defined in quantum theories[7]? Let us return to the example of a particle in the potential hole. Instead of the φ one can use the *energy-time* variables having the non-linear wave $u(t + t_0, \epsilon)$. The transformation into the space of parameters of classical trajectory, W, is preferable since the classical trajectory u is defined by the coordinates of W space completely, see also [13].

It has been shown that if a system has the sufficient set of conserved parameters, i.e. is completely integrable, then new variables, (η, ξ), form a simplectic manifold, T^*W, and all of them are quantum variables, q-numbers. The example of such a system is sin-Gordon model, where η is the momentum of soliton and ξ is the corresponding coordinate [14]. If the completely integrable system has a hidden symmetry than only a portion of new variables form the simplectic manifold, T^*W, and the others, λ, are ordinary numbers of the zero-modes space, C. The example can be Coulomb problem [15]. In it a pair $(angular\ momentum, angle)$ belongs to T^*W and the conserved hidden parameter, the length of $Runge - Lenz\ vector$, belongs to C. We have the same in the case of Yang-Mills fields with symmetry [16]. Therefore, one may expect that $W = T^*W + C$.

The transformation of l.h.s. of Eq. (4) leads to transformation of r.h.s., j, or more exactly to the splitting of j on the projection on the axes of T^*W space, $j \to (j_\eta, j_\xi)$. The variable e must be split also, $e \to (e_\eta, e_\xi)$, to conserve the form of perturbations generating operator $\hat{\mathbb{K}}$. Therefore, we have come to the theory like the Hamiltonian quantum mechanics, where $\eta = \eta(t)$ may be interpreted as a generalized momentum and $\xi = \xi(t)$ as a conjugate to η generalized coordinate of a particle and $u(\mathbf{x}; \eta, \xi, \lambda)$ defines the external potential. It must be noted that T^*W is a uniform and isotropic space in the semi-classical approximation since η are time independent quantities in that case. The quantum theory in T^*W describes fluctuations disturbing this property.

It has been demonstrated that the programm of reduction of the quantum field-theoretical problem with symmetry of the arbitrary dimension to the quantum-mechanical one,

$$u : \ \varphi(\mathbf{x}, t) \to (\eta(t), \xi(t), \lambda), \ (\eta, \xi) \in T^*W, \ \lambda \in C, \tag{5}$$

can be realized. It is important that the divergences accompanying (5) are canceled in the integral (3) since the total probability is conserved. As a result we have come to the field theory with symmetry without any divergences, like quantum mechanics, if $S(u)$ is

---

[6] It must be underlined here that the procedure of the accounting of the symmetry constraints can not be done perturbatively. One can understand the quantization as a formal language which contains the words (independent degrees of freedom) and the rules (mathematics) which explain how a sentence (result) must be deduced.

[7] One can provide following elegant geometrical interpretation of the "generalized coordinates" approach. Following the general formulation [11, 12] the set of parameters γ forms the factor space W and $u(x; \gamma)$ belongs to it completely. The mapping of multi-particle dynamics into W forms the finite-dimensional hypersurface in it. The hypersurface compactifies into the Arnold's hypertorus if the classical system is completely integrable [8]. Then one half of parameters γ is the radii, η, of the hypertorus and the other one is the angles, ξ. Description of the quantum system in terms of the collective-like variables γ means the description of random deformations of this hypersurface, i.e. of the surface of Arnold's hypertorus in the case of the integrable system.

finite. Indeed, as the result of mapping R have the same structure (3) with

$$DM = d\lambda \prod_t d\eta d\xi \delta \left(\dot{\eta} + \frac{\partial h}{\partial \xi} + j_\eta \right) \delta \left(\dot{\xi} - \frac{\partial h}{\partial \eta} - j_\xi \right), \tag{6}$$

where $h(\eta, \xi, \lambda) = H(u, \dot{u})$ and $H(u, \dot{u})$ has the structure of incident Hamiltonian. Notice that we use terms of the Lagrange formalism up to transformation (5). Next, the interactions are described by $U = U(u, e)$, where

$$e = e_\xi \frac{\partial u}{\partial \eta} - e_\eta \frac{\partial u}{\partial \xi}$$

and the perturbations are generated by

$$2\hat{\mathbb{K}} = \int dt \left\{ \frac{\delta}{\delta j_\eta} \frac{\delta}{\delta e_\eta} + \frac{\delta}{\delta j_\xi} \frac{\delta}{\delta e_\xi} \right\}. \tag{7}$$

The singularity of (5) means that this mapping is irreversible, while one must stay in W space forever. On the other hand the mapping is necessary because of the quantization rule: one must work with independent degrees of freedom. Therefore, the sense of QCD small-distance effects may be vague in the light of GCP, i.e. if QCD is realized on the zero measure, and it is possible that they need a new interpretation[8].

Moreover, there is no radiation in the field theories with symmetry. Indeed, the operator (7) on the measure (6) generates the *complete* set of contributions which does not contain the particle states[9]. In other words the field theory with symmetry does not contain even the notion of the "particle". This conclusion is rightful for an arbitrary field theory with symmetry.

5. Eventually a necessary and sufficient condition for the quantum field theory with symmetry in the light of GCP is knowledge of the maximally symmetry breaking solution, $u(\mathbf{x}; \eta, \xi, \lambda)$, of Lagrange equation (1). This problem still seeks for its solution, see also the review paper [18]. Perhaps it will be preferable from the phenomenological point of view to formulate in future the theory with symmetry directly in W space, i.e. in terms of an independent degrees of freedom.

Список литературы

[1] Manjavidze J, Sov. J. Nucl. Phys. **45** 442 (1987), hep-ph/0311196v1

[2] Leech J W, *Classical Mechanics* (Wiley, New York, 1958)

[3] R.W. Richard T. and R.L. Sells, *Elementary Modern Physics* (ISBN 0-205-06559-7, 1980)

---

[8]Physics knows such an example when a "good"theory was rejected. Weber's electrodynamics for instance perfectly describes all experimental facts, except the radiation of light.

[9]The free particle must have a definite energy and momentum in the traditional understanding of it, see [17].

[4] Dirac P A M, *The Principles of Quantum Mechanics* (Yeshiva Univ., 1964)

[5] Feynman R P and Hibbs A R *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965)

[6] Manjavidze J and Sissakian A Phys. Rep. **346**, 1 (2001)

[7] Froman N and Froman P O, *JWKB Approximation* (N.-Holand Pub. C., Amsterdam, 1965).

[8] Arnold V I *Mathematical Methods of Classical Mechanics* (Springer Verlag, New York, 1978)

[9] Edvards S F and Guliaev Y Proc. Roy. Soc., **A279**, 229 (1964)

[10] Grosche C *Path Integrals, Hyperbolic Spaces and Selberg Trace Formulae* (World Sci., 1995)

[11] S. Smale, Inv. Math. **11**, 45-64 (1970)

[12] Abraham A and Marsden J E *Foundations of Mechanics* (Benjamin/ Cummings Publ. Comp., Reading, Mass., 1978)

[13] Jackiw R, Nohl C and Rebbi C Particles and Fields: proceedings, Banff, Canada, 25 Aug. - 3 Sept., 1977, ed. D.H.Boal and A.N.Kamal (Plenum, New York, 1978); preprint BNL-23772 (1977)

[14] Manjavidze J and Sissakian A J. Math. Phys., **42**, 641 (2001)

[15] Manjavidze J J. Math. Phys., **41**, 5710 (2000)

[16] Manjavidze J and Sissakian A J. Math. Phys., **42**, 4158 (2001)

[17] Landau L and Peierls R Zs. Phys., **69**, 56 (1931)

[18] Actor A Rev. Mod. Phys., **51**, 461 (1979)